

Spontaneous Color Generation

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Quasi fermions characterized by constant vector potentials are found to be generated in a low energy effective theory of the unified model of fermions proposed in the previous paper. In reflection of the rotational symmetry of the original theory, they are effectively represented by triplets of Dirac fields with $SU(3)$ symmetry and can be regarded as quarks. Though the constant vector potentials emerge as a result of the spontaneous violation of Lorentz invariance, those are removable by suitable local phase transformations of effective fields, which implies that the emergent theory will be Lorentz invariant. The intrinsic mass of quasi quarks are found to be zero, though the vector potential gives a mass like term in the asymptotic form of the dispersion relation. The low energy effective theory reproduces also the generation structure of quarks. The number of generation depends on the cutoff scale, but is maximally 3.

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I. INTRODUCTION

This is a sequel of the first paper [1] proposing the unified picture of fermions. The model has described leptons and quarks by quasi excitations in the superconductive phase of vacuum which breaks Lorentz invariance spontaneously. In particular, the explicit calculation proves the emergence of a leptonic doublet, and the numerical estimation reveals an intimate relation between the baryon asymmetry of the Universe and the spontaneous Lorentz violation. However, the generation of a quasi fermion doublet corresponding to quarks could not be verified. This paper reports that quasi quarks actually emerge from a low energy effective theory of the proposed model.

The unified picture of fermions is based on the chiral $SU(2)$ model with massive gauge bosons. In the context of local gauge field theories, this model is unique as the unified model of fermions in the following sense.

If the unified model of fermions is ultimate, it has to generate all kinds of fermions from the least number of spinor fields. In four dimensional space-time, the group theory proves that this number can not be less than two. The minimal construction is composed of the left- and right-handed Weyl spinors. If they constitute one Dirac spinor, the model will not describe interactions other than electromagnetism, since a single Dirac spinor can accommodate only abelian gauge symmetries. On the other hand, the explanation of the baryon asymmetry of the Universe requires some interaction violating the fermion number [3–5]. If the left-handed Weyl spinor and the charge conjugate of the right-handed Weyl spinor constitute one left-handed doublet, then the $SU(2)$ gauge interaction becomes describable. In this case, however, we can not further introduce the abelian gauge interaction, due to the emergent fermion number conservation and the chiral anomaly. As a result, we come to the $SU(2)$ model with a single chiral doublet interacting only with $SU(2)$ gauge bosons. The first paper actually shows that if the $SU(2)$ gauge bosons become massive a leptonic quasi fermion doublet is generated. One remarkable feature seen from the result is that the quasi leptons emerge in the Lorentz violating phase of the vacuum. The first paper hypothesized that the quasi quarks will be generated also in some Lorentz violating phase of the vacuum, without derivation.

A serious problem naturally arising from considerations of spontaneous violation of space-time symmetries will be whether the emergent theory maintains Lorentz invariance.

In the Dirac formulation of quantum mechanics [2], physical quantities appearing in classical mechanics are decomposed into the linear operators representing physical quantities and the state vectors called bras and kets. Accordingly, two kinds of symmetry transformations are possible; one is that for a linear operator and the other is that for a state vector. A Lorentz transformation of a linear operator implies a change of the observer Lorentz frame, while that of a state vector changes the actual state of the physical system. In classical relativity, two types of transformations are called the “passive” and the “active” Lorentz transformations, respectively, and one is the inverse transformation of the other.

As a general argument, spontaneous Lorentz violation (SLV) implies that the vacuum is not invariant under Lorentz transformations, while the symmetry properties for linear operators remain intact. Consequently, the invariance of

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the system under an observer Lorentz transformation still holds even when spontaneous Lorentz violation occurs. In this sense, the emergent theory is still Lorentz invariant. However, if a quasi fermion emergent from SLV shows some property incompatible with relativity in some Lorentz frame, it would be transformed to every Lorentz frame and therefore, the breakdown of relativity would become manifest. An evidence of SLV would possibly appear in the dispersion relation of a quasi fermion. In order that no deviation from the relativistic kinematics is detectable, it seems that the dispersion relation of a quasi fermion should have the quasi-relativistic form

$$\omega = \sqrt{(\mathbf{p} + \boldsymbol{\delta})^2 + m^2} - \delta_0, \quad (1)$$

where a real constant 4-vector $\delta^\mu = (\delta_0, \boldsymbol{\delta})$ characterizes deviations from the relativistic dispersion relation. If the dispersion relation of a quasi fermion has this form, the Lorentz violating parameters δ_0 and $\boldsymbol{\delta}$ are absorbable by a suitable local phase transformation of an effective spinor field, and would not cause any physical effect. We have already seen in the first paper that the case with $\boldsymbol{\delta} = 0$ and $\delta_0 \neq 0$ actually occurs for quasi leptons. If a quasi fermion with $\boldsymbol{\delta} \neq 0$ emerges, we can expect for it some interesting characteristics.

When a quasi fermion with $\boldsymbol{\delta} = (\delta, 0, 0)$ results from SLV, those with $\boldsymbol{\delta} = (0, \delta, 0)$ and $\boldsymbol{\delta} = (0, 0, \delta)$ will also result due to the spacial symmetry of the underlying theory. These three states of a quasi fermion should be viewed as distinct fermions, since the rotational symmetry is now spontaneously broken. Accordingly, a quasi fermion with $\boldsymbol{\delta} \neq 0$ will necessarily emerge in a triplet. Though these three states originally constitute a fundamental representation of the rotational group, the state vectors can not be transformed to one another by $SO(3)$, since the spacial rotation would transform also the vacuum. If we want to express the underlying symmetry in an effective theory, it should be expressed not by the symmetry for the state vectors, but by that for their effective fields. If we represent effectively the quasi fermions with $\boldsymbol{\delta} \neq 0$ as a triplet of Dirac fields, the underlying rotational symmetry could not be expressed other than $SU(3)$, since a spinor field distinguishing a fermion from an anti fermion is essentially complex.

It is well known in the arguments of spontaneous symmetry breaking that the symmetry broken by the vacuum is maintained by the Nambu-Goldstone bosons [6, 7]. The field representations of Nambu-Goldstone bosons are obtained by localizing the transformation parameters of broken generators [8]. The same role is expected to be played in an effective theory by the gauge bosons of the localized $SU(3)$ symmetry. Then the system of quasi fermions with constant vector potentials emergent from SLV will be equivalent to QCD [9, 10].

This paper shows that SLV really generates such a constant vector potential in an low energy approximation of the unified model of fermions [1], which completes the truth of the hypothesis made in the first paper that the chiral $SU(2)$ model with massive gauge bosons would provide the quasi fermion representation for both leptons and quarks.

Moreover, the investigation in this paper can reproduce even the generation structure of quarks. The existence of quasi quarks is demonstrated by a variational method. Following the principle given in the first paper, we seek for quasi quark candidates in some Lorentz violating phase of the superconductive type vacuum [11], which is constructed from vectorial Cooper pairs and some arbitrary parameters. Simplest cases are two types of vacua, one is constituted of Cooper pairs of vector meson type (VM), and the other is of vector-dion type (VD). The vacuum in which quasi relativistic quarks are found is of type VM.

The extremum condition gives a set of self consistency equations for mass and potential parameters, which generally allow several solutions. In our case, the self consistency equations have maximally three solutions, or equivalently, three quark generations. The dispersion relations are quasi relativistic for all generations.

II. QUASI QUARKS

We consider as a low energy approximation of the unified model of fermions the following Hamiltonian with the four fermion interactions:

$$H = \int d^3x \left[\psi^\dagger i\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \psi + KV \psi^\dagger \bar{\sigma}^\mu \frac{\rho_a}{2} \psi \psi^\dagger \bar{\sigma}_\mu \frac{\rho_a}{2} \psi \right], \quad (2)$$

where K is a constant with mass dimension one, defined by $K = g^2/2m_A^2 V$. The other notations have the same meanings as given in [1]. Expressed in terms of 2-spinors, (2) is rewritten by $H = H + H'$, where

$$\begin{aligned} H_0 &= \int d^3x \left[\varphi_1^\dagger i\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \varphi_1 + \varphi_2^\dagger i\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \varphi_2 \right], \\ H' &= \int d^3x \left[\frac{1}{2} \left(\varphi_1^\dagger \bar{\sigma}^\mu \varphi_2 \varphi_2^\dagger \bar{\sigma}_\mu \varphi_1 + \varphi_2^\dagger \bar{\sigma}^\mu \varphi_1 \varphi_1^\dagger \bar{\sigma}_\mu \varphi_2 \right) + \frac{1}{4} \left(\varphi_1^\dagger \bar{\sigma}^\mu \varphi_1 - \varphi_2^\dagger \bar{\sigma}^\mu \varphi_2 \right)^2 \right]. \end{aligned} \quad (3)$$

The left-handed Weyl spinors φ_1 and φ_2 are understood as time-independent Schrödinger operators. We further introduce the annihilation operators $a_{\mathbf{pL}}$, $b_{\mathbf{pL}}$, $a_{\mathbf{pR}}$ and $b_{\mathbf{pR}}$ for primary fermions by

$$\varphi_1(\mathbf{x}) = \sum_{\mathbf{p}} (a_{\mathbf{pL}} e^{i\mathbf{p} \cdot \mathbf{x}} + b_{\mathbf{pR}}^\dagger e^{-i\mathbf{p} \cdot \mathbf{x}}) \frac{\varphi_{\mathbf{pL}}}{\sqrt{V}}, \quad \varphi_2(\mathbf{x}) = \sum_{\mathbf{p}} (b_{\mathbf{pL}} e^{i\mathbf{p} \cdot \mathbf{x}} + a_{\mathbf{pR}}^\dagger e^{-i\mathbf{p} \cdot \mathbf{x}}) \frac{\varphi_{\mathbf{pL}}}{\sqrt{V}}. \quad (4)$$

“L” and “R” denote the left-handed and the right-handed helicity states of primary fermions, respectively. We prepare as the vacuum state the following wave function:

$$|\text{VM}\rangle = \prod_{\mathbf{p}} [(\alpha_{1\mathbf{p}} + \beta_{1\mathbf{p}}(a_{\mathbf{pL}}b_{-\mathbf{pR}})^\dagger)(\alpha_{2\mathbf{p}} + \beta_{2\mathbf{p}}(b_{\mathbf{pL}}a_{-\mathbf{pR}})^\dagger)] |0\rangle, \quad (5)$$

where the complex coefficients $\alpha_{i\mathbf{p}}$ and $\beta_{i\mathbf{p}}$ satisfy the conditions

$$|\alpha_{i\mathbf{p}}|^2 + |\beta_{i\mathbf{p}}|^2 = 1, \quad (i = 1, 2). \quad (6)$$

On the other hand, the annihilation operators of quasi fermions are definable in the form

$$\begin{cases} q_{1\mathbf{p}} = \alpha_{1\mathbf{p}}a_{\mathbf{pL}} + \beta_{1\mathbf{p}}b_{-\mathbf{pR}}^\dagger \\ q_{2\mathbf{p}} = \alpha_{2\mathbf{p}}b_{\mathbf{pL}} + \beta_{2\mathbf{p}}a_{-\mathbf{pR}}^\dagger \end{cases}, \quad \begin{cases} \bar{q}_{1\mathbf{p}} = \alpha_{1-\mathbf{p}}b_{\mathbf{pR}} - \beta_{1-\mathbf{p}}a_{-\mathbf{pL}}^\dagger \\ \bar{q}_{2\mathbf{p}} = \alpha_{2-\mathbf{p}}a_{\mathbf{pR}} - \beta_{2-\mathbf{p}}b_{-\mathbf{pL}}^\dagger \end{cases}, \quad (7)$$

which annihilate the vacuum $(q_i, \bar{q}_i)|\text{VM}\rangle = 0$ and satisfy the ordinary anti-commutation relations $\{q_i, q_j^\dagger\} = \{\bar{q}_i, \bar{q}_j^\dagger\} = \delta_{ij}$ and $\{q_i, q_j\} = \{\bar{q}_i, \bar{q}_j\} = \{q_i, \bar{q}_j^\dagger\} = 0$.

Then the energy of the vacuum $W = \langle \text{VM} | H | \text{VM} \rangle$ is estimated as

$$W = \sum_{\mathbf{p}} 2|\mathbf{p}|(|\beta_{1\mathbf{p}}|^2 + |\beta_{2\mathbf{p}}|^2) + K \sum_{\mathbf{p}, \mathbf{q}} \left[\begin{aligned} & -3/2 \\ & -2\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}}(|\beta_{1\mathbf{p}}|^2|\beta_{1\mathbf{q}}|^2 + |\beta_{2\mathbf{p}}|^2|\beta_{2\mathbf{q}}|^2 + |\beta_{1\mathbf{p}}|^2|\beta_{2\mathbf{q}}|^2) \\ & -i\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}}^+ |\beta_{1\mathbf{p}}|^2 (2\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + 2\alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + \alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + \alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}) \\ & -i\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}}^+ |\beta_{2\mathbf{p}}|^2 (\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + \alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + 2\alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + 2\alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}) \\ & + \frac{1}{2}\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{e}_{\mathbf{q}}^+ \left\{ \begin{aligned} & (\alpha_{1\mathbf{p}}\beta_{1\mathbf{p}}^* + \alpha_{1-\mathbf{p}}^*\beta_{1-\mathbf{p}})(\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + \alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}}) \\ & + (\alpha_{2\mathbf{p}}\beta_{2\mathbf{p}}^* + \alpha_{2-\mathbf{p}}^*\beta_{2-\mathbf{p}})(\alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + \alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}) \\ & + (\alpha_{1\mathbf{p}}\beta_{1\mathbf{p}}^* + \alpha_{1-\mathbf{p}}^*\beta_{1-\mathbf{p}})(\alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + \alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}) \end{aligned} \right\} \end{aligned} \right], \quad (8)$$

where $\mathbf{e}_{\mathbf{p}}^\pm = \mathbf{e}_{\mathbf{p}}^1 \pm i\mathbf{e}_{\mathbf{p}}^2$ and $\mathbf{e}_{\mathbf{p}} = \mathbf{e}_{\mathbf{p}}^3$ in terms of the orthonormal basis vectors $\mathbf{e}_{\mathbf{p}}^a$. The explicit representations of $\mathbf{e}_{\mathbf{p}}^a$ are given by

$$\begin{cases} \mathbf{e}_{\mathbf{p}}^1 = (-\sin \phi, & \cos \phi, & 0), \\ \mathbf{e}_{\mathbf{p}}^2 = (-\cos \theta \cos \phi, & -\cos \theta \sin \phi, & \sin \theta), \\ \mathbf{e}_{\mathbf{p}}^3 = (\sin \theta \cos \phi, & \sin \theta \sin \phi, & \cos \theta), \end{cases} \quad (9)$$

where (θ, ϕ) are the polar coordinates of momentum \mathbf{p} . The terms $|\alpha_{i\mathbf{p}}|^2$ have been eliminated from (8) by using the constraints (6).

A. Variational equations

In order to find the ground state of the system, we minimize W with respect to the coefficients $\alpha_{i\mathbf{p}}$, $\beta_{i\mathbf{p}}$, $\alpha_{i\mathbf{p}}^*$ and $\beta_{i\mathbf{p}}^*$ under the constraints (6). Introducing Lagrange multipliers λ_i , we have a variational equation

$$\delta \left(W - \sum_{i=1}^2 \sum_{\mathbf{p}} \lambda_{i\mathbf{p}} (|\alpha_{i\mathbf{p}}|^2 + |\beta_{i\mathbf{p}}|^2 - 1) \right) = 0, \quad (10)$$

which can be written in the following form

$$\begin{cases} \frac{\partial W}{\partial \alpha_{i\mathbf{p}}^*} = A_{i\mathbf{p}}\alpha_{i\mathbf{p}} + B_{i\mathbf{p}}\beta_{i\mathbf{p}} = \lambda_{i\mathbf{p}}\alpha_{i\mathbf{p}}, \\ \frac{\partial W}{\partial \beta_{i\mathbf{p}}^*} = C_{i\mathbf{p}}\alpha_{i\mathbf{p}} + D_{i\mathbf{p}}\beta_{i\mathbf{p}} = \lambda_{i\mathbf{p}}\beta_{i\mathbf{p}}. \end{cases} \quad (11)$$

Variations with respect to $\alpha_{i\mathbf{p}}$ and $\beta_{i\mathbf{p}}$ lead no new equations. Since $A_{i\mathbf{p}} = 0$, $C_{i\mathbf{p}} = B_{i\mathbf{p}}^*$, and that $D_{i\mathbf{p}}$ are real and contain an additive term $2|\mathbf{p}|$, we can introduce new parameters $m_{i\mathbf{p}}$ and $\delta_{i\mathbf{p}}$ by

$$m_{i\mathbf{p}} = -C_{i\mathbf{p}}, \quad \delta_{i\mathbf{p}} = -|\mathbf{p}| + D_{i\mathbf{p}}/2. \quad (12)$$

The mass parameters $m_{i\mathbf{p}}$ are complex and the potential parameters $\delta_{i\mathbf{p}}$ are real. They depend on the direction of momentum \mathbf{p} , but not on the magnitude $|\mathbf{p}|$.

The Lagrange multipliers $\lambda_{i\mathbf{p}}$ should be solutions of the eigen-equation:

$$\begin{vmatrix} -\lambda_{i\mathbf{p}} & -m_{i\mathbf{p}}^* \\ -m_{i\mathbf{p}} & 2(|\mathbf{p}| + \delta_{i\mathbf{p}}) - \lambda_{i\mathbf{p}} \end{vmatrix} = 0, \quad (13)$$

and we find two solutions

$$\lambda_{i\mathbf{p}} = |\mathbf{p}| + \delta_{i\mathbf{p}} \pm \omega_{i\mathbf{p}}, \quad \omega_{i\mathbf{p}} = \sqrt{(|\mathbf{p}| + \delta_{i\mathbf{p}})^2 + |m_{i\mathbf{p}}|^2}. \quad (14)$$

The introduction of $m_{i\mathbf{p}}$ and $\delta_{i\mathbf{p}}$ rewrites the coefficients $\alpha_{i\mathbf{p}}$ and $\beta_{i\mathbf{p}}$ as

$$\alpha_{i\mathbf{p}} = \sqrt{\frac{1}{2} \left(1 \mp \frac{|\mathbf{p}| + \delta_{i\mathbf{p}}}{\omega_{i\mathbf{p}}} \right)}, \quad \beta_{i\mathbf{p}} = \mp \frac{m_{i\mathbf{p}}}{|m_{i\mathbf{p}}|} \sqrt{\frac{1}{2} \left(1 \pm \frac{|\mathbf{p}| + \delta_{i\mathbf{p}}}{\omega_{i\mathbf{p}}} \right)}, \quad (15)$$

in the phase convention: $\alpha_{i\mathbf{p}}^* = \alpha_{i\mathbf{p}}$. The selection of the lower signs in (15) minimizes W , since otherwise $|\beta_{i\mathbf{p}}| \rightarrow 1$ when $|\mathbf{p}| \rightarrow \infty$, which implies in (8) that high-momentum fermions would dominate the vacuum.

The equations (11) then give the self consistency equations for $m_{i\mathbf{p}}$ and $\delta_{i\mathbf{p}}$:

$$\begin{cases} m_{1\mathbf{p}} = K \sum_{\mathbf{p}} & i\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{e}_{\mathbf{q}} (2|\beta_{1\mathbf{q}}|^2 + |\beta_{2\mathbf{q}}|^2) \\ & -\frac{1}{2}\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{e}_{\mathbf{q}}^+ (2\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + 2\alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + \alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + \alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}), \\ \delta_{1\mathbf{p}} = K \sum_{\mathbf{p}} & -\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}} (2|\beta_{1\mathbf{q}}|^2 + |\beta_{2\mathbf{q}}|^2) \\ & -\frac{i}{2}\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}}^+ (2\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + 2\alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + \alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + \alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}), \end{cases} \quad (16)$$

$$\begin{cases} m_{2\mathbf{p}} = K \sum_{\mathbf{p}} & i\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{e}_{\mathbf{q}} (|\beta_{1\mathbf{q}}|^2 + 2|\beta_{2\mathbf{q}}|^2) \\ & -\frac{1}{2}\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{e}_{\mathbf{q}}^+ (\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + \alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + 2\alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + 2\alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}), \\ \delta_{2\mathbf{p}} = K \sum_{\mathbf{p}} & -\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}} (|\beta_{1\mathbf{q}}|^2 + 2|\beta_{2\mathbf{q}}|^2) \\ & -\frac{i}{2}\mathbf{e}_{\mathbf{p}} \cdot \mathbf{e}_{\mathbf{q}}^+ (\alpha_{1\mathbf{q}}\beta_{1\mathbf{q}}^* + \alpha_{1-\mathbf{q}}^*\beta_{1-\mathbf{q}} + 2\alpha_{2\mathbf{q}}\beta_{2\mathbf{q}}^* + 2\alpha_{2-\mathbf{q}}^*\beta_{2-\mathbf{q}}). \end{cases} \quad (17)$$

We see from (16) and (17) that $m_{i\mathbf{p}}$ and $\delta_{i\mathbf{p}}$ are expressible in the forms : $m_{i\mathbf{p}} = i\mathbf{e}_{\mathbf{p}}^+ \cdot \mathbf{m}_i$ and $\delta_{i\mathbf{p}} = \mathbf{e}_{\mathbf{p}} \cdot \boldsymbol{\delta}_i$, where \mathbf{m}_i and $\boldsymbol{\delta}_i$ are real constant 3-vectors. Then we further see that $\mathbf{m}_i = -\boldsymbol{\delta}_i$, and consequently, $\omega_{i\mathbf{p}} = |\mathbf{p} + \boldsymbol{\delta}_i|$ hold.

The self consistency equations which two constant vector potentials $\boldsymbol{\delta}_i$ should satisfy are reduced to

$$\begin{cases} \boldsymbol{\delta}_1 = K \sum_{\mathbf{q}} \frac{\mathbf{q} + \boldsymbol{\delta}_1}{|\mathbf{q} + \boldsymbol{\delta}_1|} + \frac{1}{2} \frac{\mathbf{q} + \boldsymbol{\delta}_2}{|\mathbf{q} + \boldsymbol{\delta}_2|}, \\ \boldsymbol{\delta}_2 = K \sum_{\mathbf{q}} \frac{1}{2} \frac{\mathbf{q} + \boldsymbol{\delta}_1}{|\mathbf{q} + \boldsymbol{\delta}_1|} + \frac{\mathbf{q} + \boldsymbol{\delta}_2}{|\mathbf{q} + \boldsymbol{\delta}_2|}. \end{cases} \quad (18)$$

The introduction of a 3-momentum cut off Λ estimates the momentum sum as

$$K \sum_{\mathbf{q}} \frac{\mathbf{q} + \boldsymbol{\delta}}{|\mathbf{q} + \boldsymbol{\delta}|} = \frac{4}{3} \xi^2 \boldsymbol{\delta} \left(1 - \frac{\boldsymbol{\delta}^2}{5\Lambda^2} \right), \quad \xi = \frac{g\Lambda}{4\pi m_A}. \quad (19)$$

The form of equations (18) show that two vector potentials $\boldsymbol{\delta}_i$ are collinear and expressible in the form $\boldsymbol{\delta}_i = \delta_i \boldsymbol{\epsilon}$ with the help of a unit vector $\boldsymbol{\epsilon}$ in the common direction. Adding and subtracting two equations of (18) we have the equations for δ_i :

$$\begin{cases} (\delta_1 + \delta_2) \left[\frac{1}{2\xi^2} - 1 + \frac{1}{5\Lambda^2} (\delta_1^2 + \delta_2^2 - \delta_1\delta_2) \right] = 0, \\ (\delta_1 - \delta_2) \left[\frac{1}{2\xi^2} - 1 + \frac{1}{5\Lambda^2} (\delta_1^2 + \delta_2^2 + \delta_1\delta_2) \right] = 0. \end{cases} \quad (20)$$

TABLE I: The number of generations G for quasi quarks

ξ	0	$1/\sqrt{2}$	$\sqrt{3/2}$	$\sqrt{2}$	$+\infty$
G	0	1	2	3	

The multitude of solutions for the algebraic equations (20) depends on the magnitude of ξ , and is at most 3. These are

$$\begin{aligned}
G_1 : \delta_1 &= \delta_2 = \pm \frac{4\pi\sqrt{5}}{g} m_A \sqrt{\xi^2 - 1/2}, \\
G_2 : \delta_1 &= -\delta_2 = \pm \frac{4\pi\sqrt{5}}{g} m_A \sqrt{\xi^2 - 3/2}, \\
G_3 : (\delta_1, \delta_2) \text{ or } (\delta_2, \delta_1) &= \left(\pm \frac{2\pi\sqrt{5}}{g} m_A (\xi + \sqrt{\xi^2 - 2}), \mp \frac{2\pi\sqrt{5}}{g} m_A (\xi - \sqrt{\xi^2 - 2}) \right).
\end{aligned} \tag{21}$$

We do not count the solutions obtained by reversing the signs or interchanging the names of δ_i as independent. The solution G_1 is present for $\xi > 1/\sqrt{2}$, G_2 for $\xi > \sqrt{3/2}$, and G_3 for $\xi > \sqrt{2}$.

B. Dispersion relations, color charges and the number of generation

We calculate the matrix elements of the Hamiltonian with the one-quasi-quark states:

$$\langle q_a H q_b^\dagger \rangle = \begin{bmatrix} |\mathbf{p} + \boldsymbol{\delta}_1| & 0 & 0 & 0 \\ 0 & |\mathbf{p} + \boldsymbol{\delta}_2| & 0 & 0 \\ 0 & 0 & |\mathbf{p} - \boldsymbol{\delta}_1| & 0 \\ 0 & 0 & 0 & |\mathbf{p} - \boldsymbol{\delta}_2| \end{bmatrix}, \tag{22}$$

where q_a runs over $q_{1\mathbf{p}}$, $q_{2\mathbf{p}}$, $\bar{q}_{1\mathbf{p}}$, and $\bar{q}_{2\mathbf{p}}$. The above expression shows that the one-particle states $q_{i\mathbf{p}}^\dagger |\text{VM}\rangle$ obtained by the variational method are actually the one-particle energy eigenstates of the Hamiltonian. Each expression of energies is quasi relativistic, with the intrinsic mass $m = 0$ and with the scalar (time-like) potential $\delta_0 = 0$. Despite of zero intrinsic mass, the asymptotic form of dispersion relation at high momentum:

$$\omega = |\mathbf{p} + \boldsymbol{\delta}| \simeq |\mathbf{p}| + |\boldsymbol{\delta}| \cos \theta + \frac{(|\boldsymbol{\delta}| \sin \theta)^2}{2|\mathbf{p}|}, \tag{23}$$

indicates that a quasi quark has an effective mass $m_{\text{eff}} = |\boldsymbol{\delta}| \sin \theta$, where θ is the angle which $\boldsymbol{\delta}$ makes with \mathbf{p} . The effective mass is direction dependent; the longitudinal mass is zero, while the transversal mass is $|\boldsymbol{\delta}|$.

Another feature recognizable from (22) is that the vector potential of a quasi anti fermion has the opposite direction to that of a quasi fermion. If we denote quasi fermions with $\boldsymbol{\delta} = (\delta, 0, 0)$, $(0, \delta, 0)$ and $(0, 0, \delta)$ by q_R , q_G and q_B , respectively, then the quasi anti fermions \bar{q}_R , \bar{q}_G and \bar{q}_B have $\boldsymbol{\delta} = (-\delta, 0, 0)$, $(0, -\delta, 0)$ and $(0, 0, -\delta)$, respectively. The similar feature has already been seen in the first paper [1], which played the essential role to generate the baryon asymmetry of the Universe [12, 13].

Quasi fermions with different vector potentials are present on different vacua. Though they are distinct particles, they are originally related by rotations. The distinction comes from the violation of rotational symmetry of the vacua. However, according to the property of spontaneous symmetry breaking, the broken symmetry is maintained by the emergence of Nambu-Goldstone bosons. Therefore, q_R , q_G and q_B will be transformed to one another by mediations of the Nambu-Goldstone bosons emergent from spontaneous violation of rotational symmetry. If an effective field theory assigns to (q_R, q_G, q_B) a triplet of Dirac fields, the original rotational symmetry will be realized by SU(3) transformations. Then the role of Nambu-Goldstone bosons will be played effectively by the SU(3) gauge bosons. The property of infrared slavery of SU(3) gauge interactions is thought to explain the quark confinement [14, 15]. Consequently, the quasi fermions with non-zero constant vector potentials $\boldsymbol{\delta}_i$ will be well qualified to be regarded as quarks.

The number of independent solutions of (20), which is the same as the number of generations, increases with increasing ξ as shown in Table I. The maximum number of generations is 3, or equivalently, the maximum number of quark flavors is 6.

III. SUMMARY AND CONCLUSIONS

We have sought the quasi particle representation of quarks in the context of the unified picture of fermions [1], and have succeeded to show the explicit construction of quasi quarks lacking in the first paper. This result completes the proof that the unified model of fermions actually deserves its name.

We can recognize from the results of the first and this paper that the typical dispersion relation of a quasi fermion appearing from spontaneous Lorentz violation has the quasi relativistic form including a constant 4-vector potential $\delta^\mu = (\delta_0, \boldsymbol{\delta})$. The origin of distinction between leptons and quarks stems from the constant vector potential $\boldsymbol{\delta}$. A quasi fermion with $\boldsymbol{\delta} = \mathbf{0}$ becomes a lepton, while that with $\boldsymbol{\delta} \neq \mathbf{0}$ becomes a quark. Thus the unified picture of fermions can explain why there appear two distinct types of elementary fermions in nature.

We examined quasi excitations of the vacuum of type VM, and found that quasi fermions well reproduce the nature of real quarks, including the number of generations. The SU(3) interaction in QCD is understandable as an effective field realization of Nambu-Goldstone bosons originating from the broken SO(3) symmetry.

Our analysis in this paper is based on a four fermion approximation of the original theory (2). The validity of this approximation seems to restrict the energy scale to be lower than m_A . If this upper limit applies to the cutoff scale Λ , then ξ should be nearly less than $1/20$. However, we have seen that quasi quarks require larger values, $\xi \simeq 1$, which may give rise to a question for the validity of the results obtained here. We have seen in the first paper that the quasi leptons require $\xi > 1$. Therefore, the value $\xi \simeq 1$ required for quasi quarks seems to indicate the characteristics of the original theory, not influenced by the four fermion approximation.

The emergence of both quasi leptons and quasi quarks from the unified model of fermions strengthens further the view suggested in the first paper that the structure of the standard model as the laws in the present Universe originates from the occurrence of spontaneous Lorentz violation by fermions at some epoch of the early Universe, though the generation structure of leptons and an effective field realization of the electromagnetic interactions remain yet to be understood.

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